Newtonian Gravitation Theory:

Two-Body Orbital Systems (Figure 1):

For any 2 masses, in orbital motion

(1) $$\mathbf{F_g} = \frac{\mathbf{G \cdot m_1 \cdot m_2}}{\mathbf{R}^2}$$

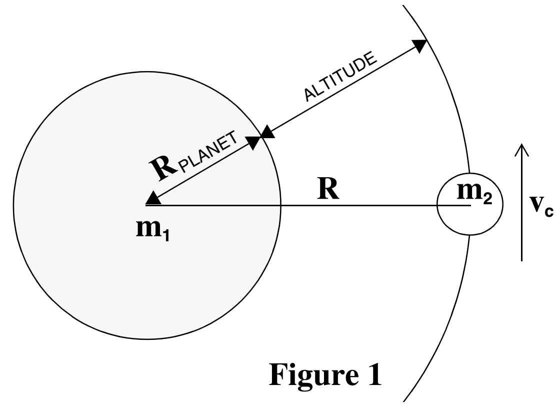

Figure 1

For any mass in circular motion

(2) $$\mathbf{F_c} = \mathbf{m}_2 \cdot \mathbf{a_c} = \mathbf{m}_2 \cdot \frac{\mathbf{v_c}^2}{\mathbf{R}} = \mathbf{m}_2 \cdot \frac{4\pi^2\mathbf{R}}{\mathbf{T}^2}$$

with the last expression obtained by substituting the **circular speed equation**:

(3) $$\mathbf{v_c} = \frac{2\pi\mathbf{R}}{\mathbf{T}}$$

The Newtonian Synthesis states that $\mathbf{F_g} = \mathbf{F_c}$, allowing us to combine Equ. 1 & 2 as

(4) $$\frac{\mathbf{G \cdot m_1 \cdot m_2}}{\mathbf{R}^2} = \mathbf{m}_2 \cdot \frac{4\pi^2\mathbf{R}}{\mathbf{T}^2}$$

Solving Equ. 4 for selected variables, we have

(5) $$\mathbf{T} = \sqrt{\frac{4\pi^2\mathbf{R}^3}{\mathbf{Gm}_1}}$$

(6) $$\mathbf{R} = \sqrt[3]{\frac{\mathbf{Gm}_1\mathbf{T}^2}{4\pi^2}}$$

(7) $$\mathbf{m}_1 = \frac{4\pi^2}{\mathbf{G} \cdot \left(\frac{\mathbf{T}^2}{\mathbf{R}^3}\right)} = \frac{4\pi^2\mathbf{R}^3}{\mathbf{G \cdot T}^2}$$

Note: Equ. 7 in its unsimplified form shows the Kepler Constant (bracketed term) to be a function of the mass of the orbit<u>ed</u> body, and constant for all orbit<u>ing</u> bodies in the system.

Comparing 2 Points in a Gravitational Field (Figure2):

(8) $$\frac{\mathbf{F'_g}}{\mathbf{F_g}} = \frac{\mathbf{a'_g}}{\mathbf{a_g}} = \frac{\mathbf{g'}}{\mathbf{g}} = \left(\frac{\mathbf{R}}{\mathbf{R'}}\right)^2$$

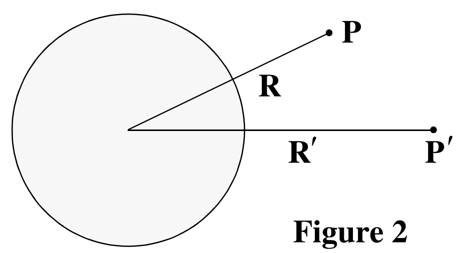

Figure 2

Additional Relevant Equations:

(9) gravitational field intensity: $\mathbf{g} = \mathbf{a_g} = \dfrac{\mathbf{Gm}}{\mathbf{R}^2}$

(10) escape velocity at distance "**R**" in gravitational field "$\mathbf{a_g}$": $\mathbf{v_e} = \sqrt{2\mathbf{Ra_g}}$

(11) for bodies in orbit around **m,** the "Kepler Constant" = $\dfrac{\mathbf{T}^2}{\mathbf{R}^3} = \dfrac{4\pi^2}{\mathbf{G \cdot m}}$

(12) density of a spherical planet: $\rho = \dfrac{\mathbf{m}}{\mathbf{V}} = \dfrac{\mathbf{m}}{\frac{4}{3}\pi\mathbf{R}^3}$