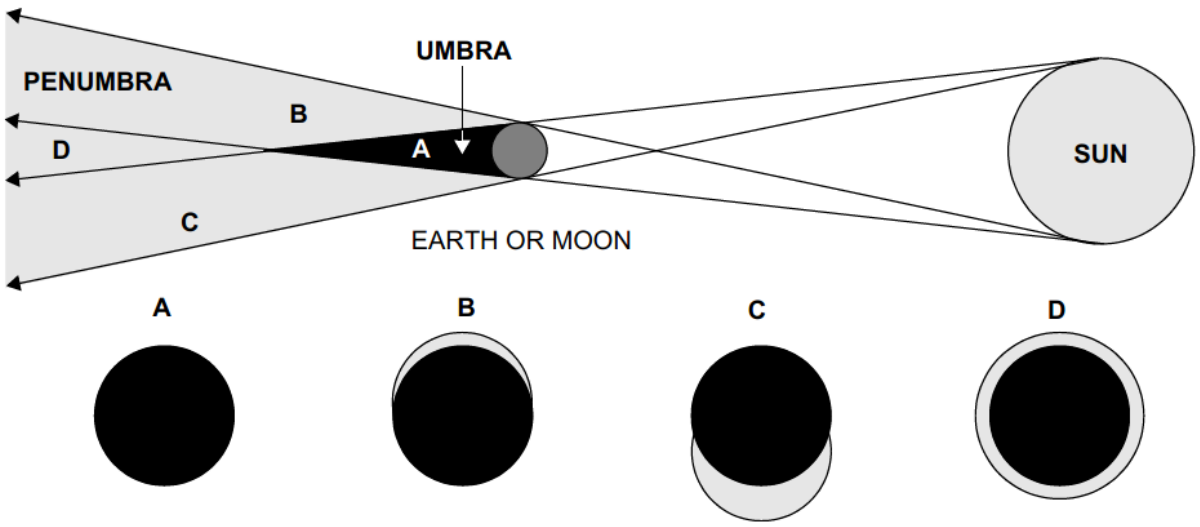
The Geometry of Solar and Lunar Eclipses:

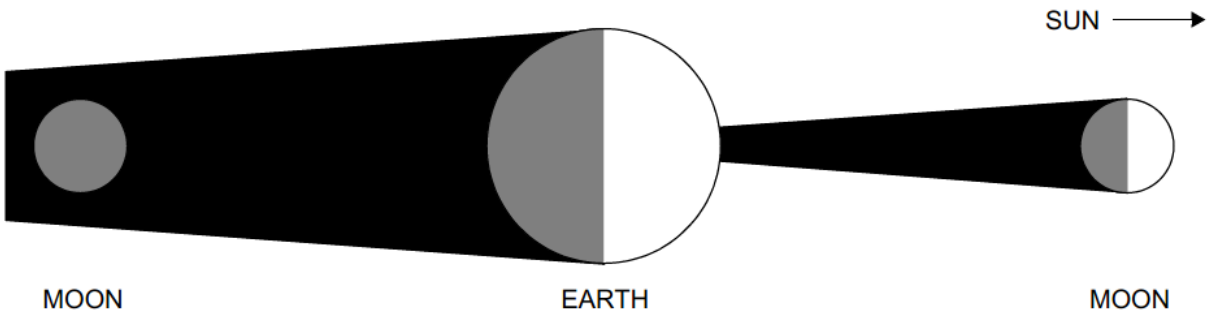
Solar eclipses occur when the shadow cast by the Moon falls on an Earthbound observer. The shadow is much smaller than the Earth, so you have to be at the right place at the right time to see one. A solar eclipse can only occur during a New Moon.

Lunar eclipses occur when the Moon moves through the shadow cast by the Earth. Anybody on the night side of Earth will be able to see this when it happens, as the (necessarily) Full Moon moves into Earth's shadow.

The diagram below shows the geometry of shadow formation when the source of light is larger than the object casting the shadow. The graphics below show what would be seen by an observer located at points A B C D.



The following diagram shows the geometry of solar and lunar eclipses (not to scale). The Sun is far off to the right outside the frame. In this example, the Moon's umbra reaches Earth's surface and an observer in that shadow would see view A above. In real eclipses, sometimes the umbra falls short and observers see an annular eclipse as portrayed in view D above.



Questions:

You will need only plane geometry, plus some astronomical data easily found online, to answer the following questions. Approximate answers are provided to help you check your calculations. Be sure to list your data, cite data sources, construct labeled diagrams, and show all your work. Submit your analysis on a separate sheet of paper with the usual identifying information.

1a. For average orbital distances, calculate the diameter of the umbra of the Earth where the Moon passes through it. [Answer: ≈ 9170 km]

lb. Given the Moon's average speed through space, for how long will the Moon will be at least partly in the umbra? [Answer: ≈ 1.7 hours]

2. For the maximum total solar eclipse, the Moon must be at its orbital perigee (closest to Earth). What is the maximum size of the Moon's shadow on Earth's surface? [Answer: ≈ 265 km]

3. How far from Earth does the Moon have to be for an annular eclipse to occur? Note this is the same question as asking "What is the length of the Moon's umbra?" [Answer: ≈ 380,000 km]

4a. As Earth rotates, an observer at its equator is being carried to the east at about 0.46 km/s. The Moon's shadow moves across the Earth from west to east at the same speed the Moon moves through space. What is the speed of the shadow relative to an observer? [Answer: ≈ 0.55 km/s]

4b. What is the maximum duration of a total solar eclipse? [Answer: ≈ 480 seconds]

4c. How long would the eclipse last if you "chased" the shadow in a high speed jet at a speed of v = 0.3 km/s? [Answer: ≈ 1060 seconds]

4d. If you were "watching" a solar eclipse through a telescope from the Moon, and if the Moon's shadow's path on Earth is the longest possible, how long would it take to cross the entire planet? [Answer: ≈ 3.5 hours]

5a. What is the average angular size of the Moon as seen from Earth? This is equivalent to asking "What is the vertex angle of the Moon's umbra?" [Answer: ≈ 0.5°]

5b. What is the average angular size of the Sun as seen from Earth? [Answer: ≈ 0.5°]

5c. Why are the answer for 5a and 5b the same (or nearly so)? Is this just a coincidence, or is there some underlying scientific reason for the equality? [Answer: open discussion]